

**Year 12 Examination, 2020**

**Question/Answer Booklet**

**MATHEMATICS SPECIALIST**

**Section One: Calculator-free**

Student Name/Number: \_\_\_\_\_

Teacher Name: \_\_\_\_\_

**Time allowed for this section**

Reading time before commencing work: five minutes

Working time for this section: fifty minutes

**Materials required/recommended for this section**

**To be provided by the supervisor:** This Question/Answer Booklet  
Formula Sheet

**To be provided by the candidate:**

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

### Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	11	11	100	100	65
					100

### Instructions to candidates

- The rules for the conduct of School exams are detailed in the \_\_\_\_\_ *School/College assessment policy*.  
Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
- Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you do not use pencil, except in diagrams.
- Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- The Formula sheet is not to be handed in with your Question/Answer booklet.

**Section One: Calculator-free**

**35% (50 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

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**Question 1**

**(4 marks)**

Determine the complex number  $z$  if

$$\frac{z}{\sqrt{3}-i} = \frac{1}{\sqrt{3}+i}.$$

Give your answer in both Cartesian and polar forms.

**Question 2****(9 marks)**

Consider the system of linear equations

$$2x + 3y + 2z = 10$$

$$3x + 4y - z = 4$$

$$x + y + az = b$$

where  $a$  and  $b$  are constants.

- (a) For which values of the constants  $a$  and  $b$  are there infinitely many solutions? (4 marks)

- (b) For which values of the constants  $a$  and  $b$  is there no solution? (2 marks)

- (c) Solve the system of equations when  $a = 2$  and  $b = -5$ . (3 marks)

**Question 3**

**(6 marks)**

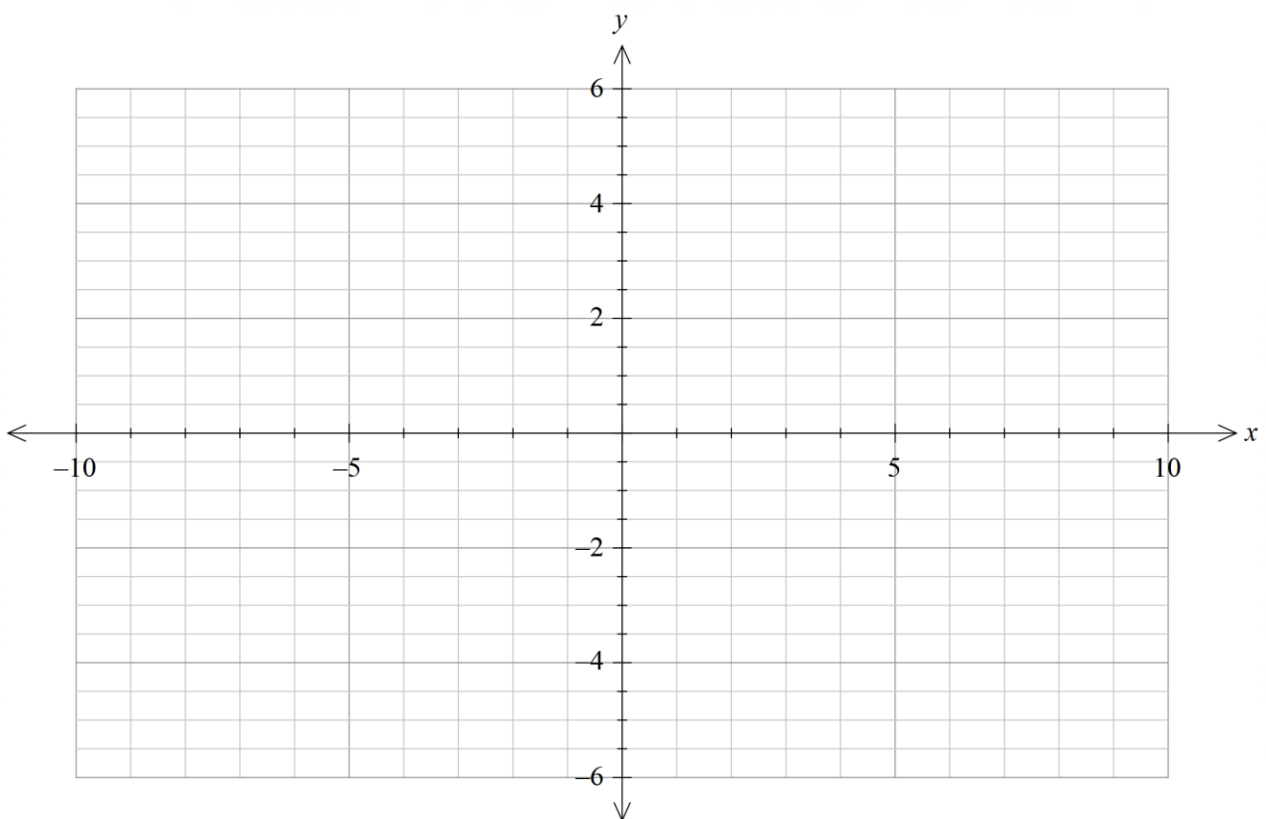
Suppose that  $f(x) = \sqrt{4 - |3 - 2x|}$ .

(a) For which real numbers  $x$  is  $f(x)$  defined?

**(3 marks)**

(b) Sketch on the axes below the graph of  $y = f(x)$ .

**(3 marks)**



**Question 4**

**(5 marks)**

The equation of a sphere S is  $x^2 + y^2 + z^2 - 4x + 10z = 20$ .

(a) Determine the radius and the co-ordinates of the centre of S. (2 marks)

(b) Determine the equation of the plane which contains the point A (0,3,1) and which is tangent to S. (3 marks)

## Question 5

(5 marks)

(a) Prove that if  $z$  is any complex number then  $z\bar{z} = |z|^2$ .

(2 marks)

(b) Prove that if  $a$  and  $b$  are any two complex numbers then

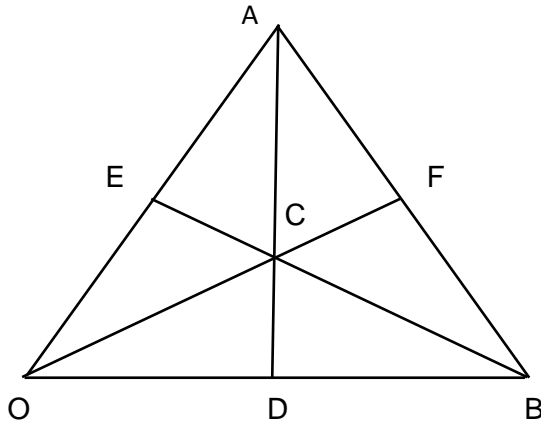
(3 marks)

$$|a+b|^2 + |a-b|^2 = 2\{|a|^2 + |b|^2\}.$$



Question 6

(6 marks)



In the triangle AOB the line segments AD, BE and OF are medians: each joins a vertex of the triangle to the mid-point of the opposite side. Denote  $\mathbf{a} = \overrightarrow{OA}$  and  $\mathbf{b} = \overrightarrow{OB}$ .

- (a) Determine, in terms of  $\mathbf{a}$  and  $\mathbf{b}$ , the vector equation of the median line that passes through B and E. (2 marks)

- (b) Show that  $\overrightarrow{OC} = \frac{1}{3}(\mathbf{a} + \mathbf{b})$ . (4 marks)

**Question 7****(7 marks)**

Determine the seven roots of the equation

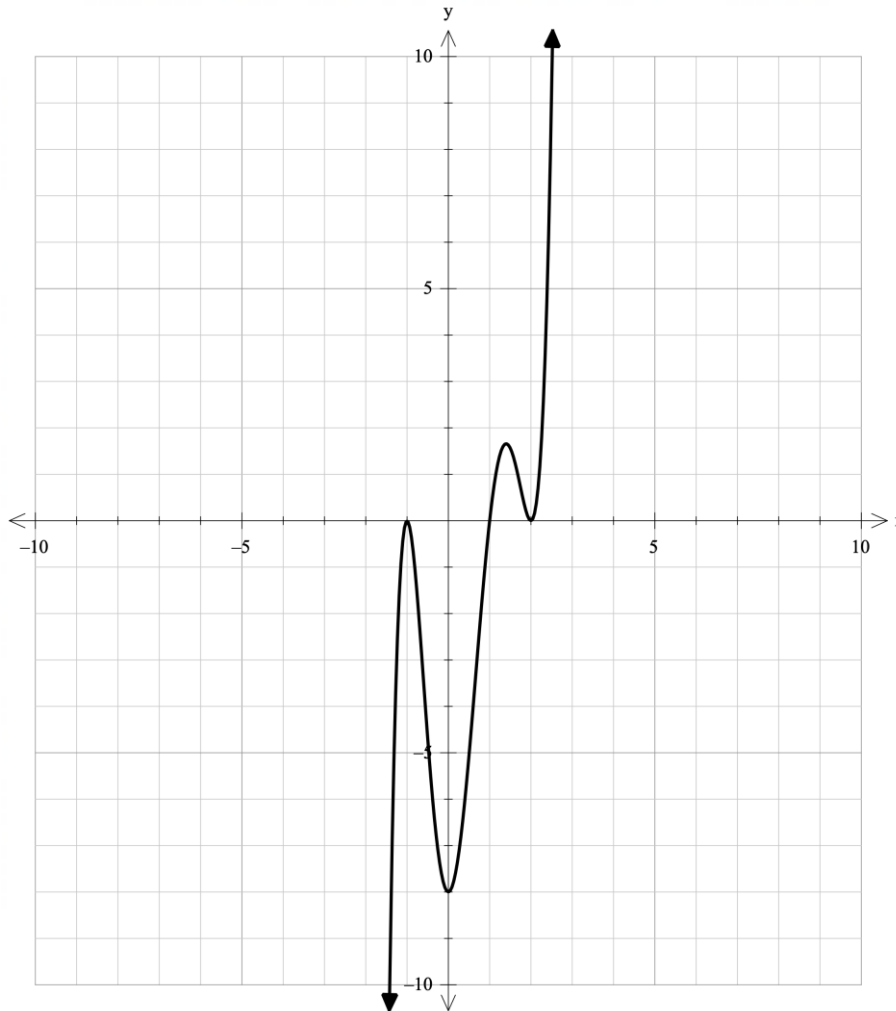
$$z^7 = -64\sqrt{2}(1+i)$$

giving the answers in polar form with arguments  $\theta$  lying in the range  $-\pi \leq \theta < \pi$ .

Question 8

(8 marks)

The graph of  $y = f(x)$  is shown below.



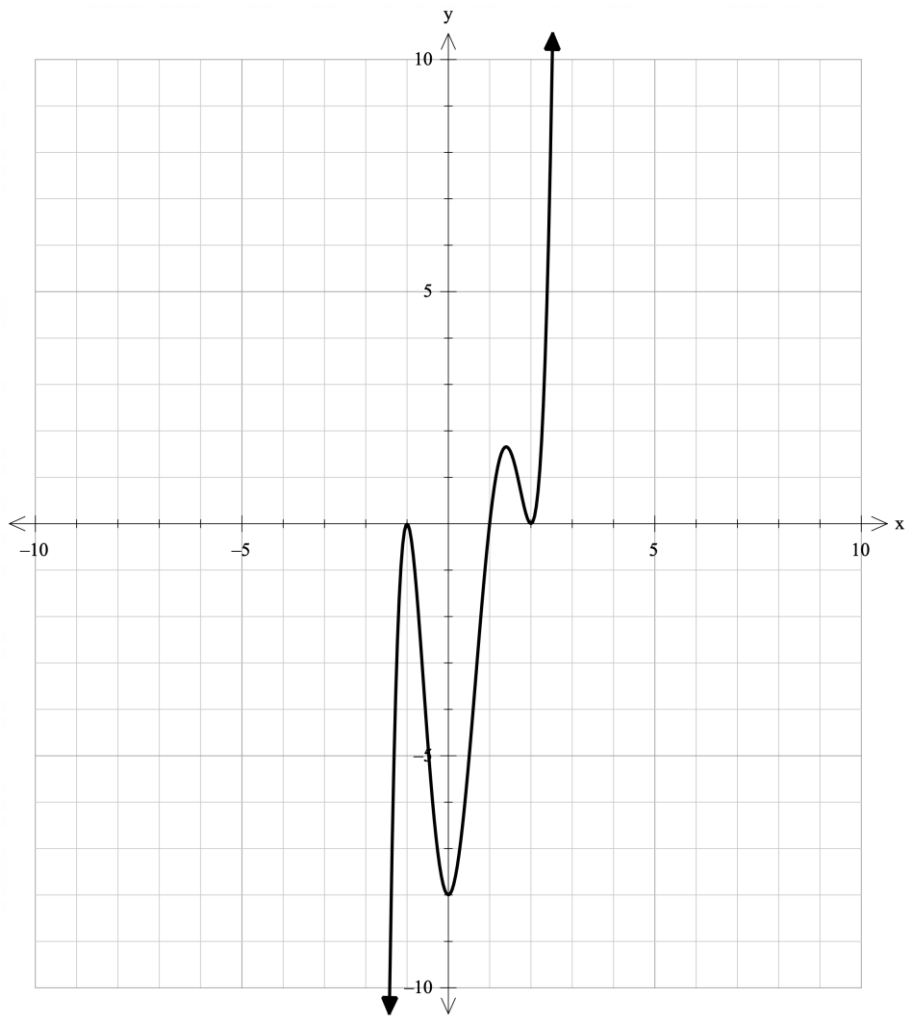
Sketch on the same axes the graph of

(a)  $y = f(x - 2)$

(2 marks)

Question 8 continues overleaf....

Question 8 continued....



Sketch on the axes above the graph of

(b)  $y = f(|x|)$

(2 marks)

- (c) The formula for  $f(x)$  is of the form (4 marks)

$$f(x) = a(x+p)^b(x-q)(x-r)^c$$

Determine the values of the six constants  $a, b, c$  and  $p, q$  and  $r$ .

**END OF QUESTIONS**

**Additional working space**

Question number: \_\_\_\_\_

**Additional working space**

Question number: \_\_\_\_\_

## Acknowledgements

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